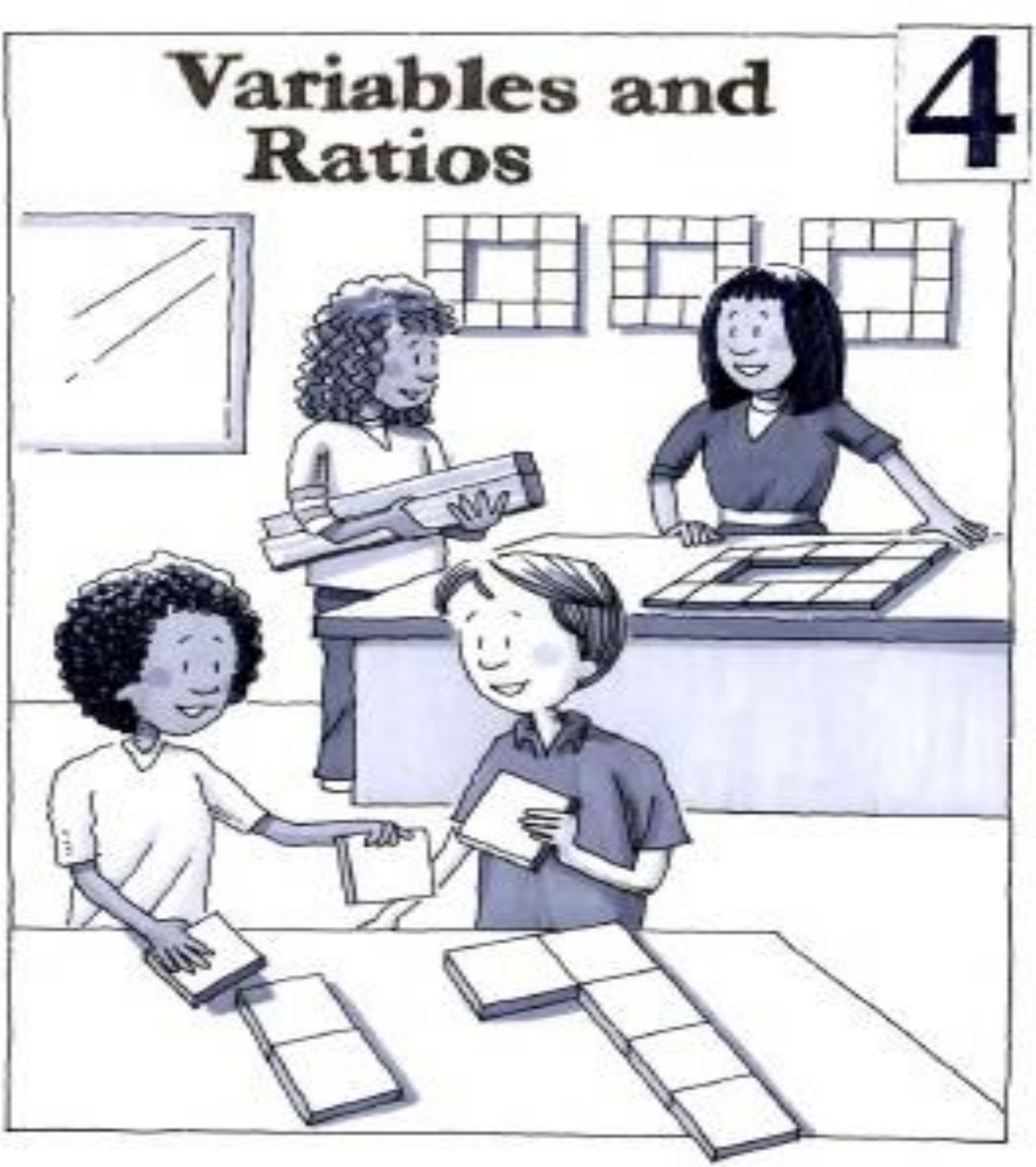
Name ________________________________ Period______ Teacher______________



Chapter 4   Variables and Ratios

Guiding Questions

Think about these questions throughout this chapter:

How can I represent it?

How can I use a variable?

What are expressions and equations?

How can I change the size but keep the shape the same?

How is it the same or different?

Are you ready to strengthen your pre-algebra skills? One skill that is essential for algebra is figuring out unknown amounts. In Section 4.1, you will begin to think about how to do so. You will use variables to represent unknown quantities and will use what you know about a problem to find the value of these variables.

In Section 4.2, you will move from mystery numbers to a mystery mascot. With your class, you will work to enlarge the mystery mascot. Then you will learn how to enlarge or reduce figures while keeping their shapes the same. You will use ratios to compare the side lengths of figures to determine if they are the same shape.

In this chapter, you will learn how to:

- Use variables to generalize and to represent unknown quantities.
- Write multiple expressions to describe a pattern and recognize whether the expressions are equivalent.
- Find the value of an algebraic expression when the value of the variable is known.
- Enlarge and reduce figures while maintaining their shapes.
- Use ratios to describe relationships between similar shapes.

4.1.1 What if I do not know a length?

Introduction to Variables

In the last chapter, you worked with lengths, moving back and forth on a number line, and comparing signed numbers (+ and −). But what if there are lengths you *do not* know? In this lesson, you will use clues to find unknown values. Unknown values are often represented by **variables**. Finding unknown values is one of the most important parts of algebra. Today's work will give you the background you will need for your upcoming work with variables. As you work with your team today, keep these questions in mind:

How can I represent or visualize this situation?

What information *do* I know?

What information do I need to find?

4-1. CROAKIE THE TALENTED FROG

Croakie is a very talented frog. He does tricks for the audiences at the Calaveras County Fair contest every year. Some of his tricks are quickly making him famous. He not only hops, but he can also do a "hip hop" jump, along with other exciting tricks. Just how long is his "hip hop" jump, assuming he travels the exact same distance each time? Read the description of his special routine below. Then complete parts (a) through (d) that follow.

- Croakie starts at point A. He hops 12 feet to the right, toward point B.
- Then he does two "hip hop" jumps in a row, still traveling to the right.
- He turns and makes a 3-foot hop to the left.
- He stops to regain his balance and then, still traveling to the left, repeats his 3-foot hop three more times.
- He turns and makes 16 spinning hops that are 1 foot each to the right, ending exactly at point B.

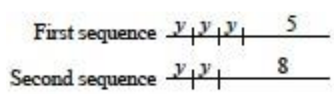
a. Draw a diagram to show Croakie's entire routine as described above.

b. Work with your team to write an expression that represents the distance from point A to point B based on Croakie's moves.

c. Jill is one of Croakie's biggest fans. From watching his act, she estimates that his "hip hop" jumps are each 5 feet long. If Jill is correct, how far is it from point A to point B? Explain.

d. Croakie's manager measured the distance from point A to point B and found that it was actually 24 feet. How far does Croakie really travel each time he does his "hip hop" jump? Use pictures to help explain your thinking. Be prepared to share your thinking with the class.

4-2. Now Croakie has a new special jump length. He moved between two fixed points, each time with a different sequence. His trainer, Thom, drew the diagram below to represent his two sequences, using y to represent the length of Croakie's new special jump.



a. Describe each of Croakie's two sequences.

b. Work with your team to figure out how far Croakie travels in each special jump. Be prepared to explain your thinking to the class.

c. What is the distance between the start and end of his sequence of jumps?

4-3. Croakie has a new set of moves. The sequence involves three special high hops. The expression $x + x + x + 5$ represents the whole sequence, with x representing the distance he moves with each high hop.

a. In your own words, describe what you know about Croakie's new sequence.

b. If Croakie's new sequence is a total of 11 feet, draw a diagram to represent Croakie's new sequence.

c. How far does Croakie jump with each high hop? How can you tell?

4-4. Lanaya is a gymnast and is working on a new routine. For her new routine, she starts by walking 4 feet to the right. Then she does one handspring, then a cartwheel, followed by a somersault, and then two more handsprings. Lanaya is very consistent and travels the same distance for each handspring. Use the details of her new routine to complete parts (a) and (b) below.

a. Work with your team to draw a diagram of Lanaya's routine. Then write an expression to represent how far Lanaya travels during her routine.

b. If Lanaya moves 6 feet during a handspring, 3 feet during a cartwheel, and 2 feet during a somersault, what distance does she cover during her routine?

4-5. Croakie is certainly a remarkable frog. Now he has developed even more amazing tricks! This time, he starts at point A, slides 2 feet to the right, and then completes two flips in a row, landing at point B. From point B, he turns around and goes back by doing one flip and sliding 8 feet to the left, ending up back at point A.

a. How far does Croakie move during each somersault, assuming each somersault is exactly the same length? Explain how you got your answer.

b. What is the distance between points A and B?

4-6. Additional Challenge: Create a new problem to challenge your teammates.

Make up a new trick for Croakie, but do not tell anyone how much distance it covers.

Design two or more different sequences that Croakie can do with his new trick while performing routines that are *the same length*. (You get to use any length you want, but again, do not tell anyone.)

Write down all of the necessary clues and be ready to trade problems with a team member.

METHODS AND MEANINGS

MATH NOTES

DIVIDING

```
    37
6)225
 -180
   45
  -42
    3
```

When using long division to divide one number by another, it is important to be sure that you know the place value of each digit in your result.

In the example of dividing 225 by 6 at right, people often begin by saying, *"6 goes into 22 three times."* If they were paying attention to place value, they would instead say *"6 goes into 220 thirty-something times."* The 3 of the quotient is written in the tens place to indicate that 6 goes into 225 at least 30 times, but less than 40. The 3 represents 3 tens.

```
    37.5
6)225.0
 -180
   45
  -42
    30
  - 30
     0
```

It may seem like the divisor is then multiplied by the 3, and the product, 18, is placed below a 22. However, you are really multiplying 30 by 6 and the product is 180, which is placed below 225. You would then subtract, getting what looks like 4. But then you would "bring down" the 5, and get 45. Notice that if you subtract 180 from 225, as in the top example at right, you get 45 directly. You then repeat the same process. In the past, you may have stopped at this point and written that the quotient is 37 with a remainder of 3.

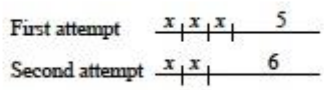
The same method works for dividing decimals. The bottom example at right is essentially the same as the top one, except that it shows what happens if you keep dividing past the decimal point, while still keeping place value in mind.

4-7. Croakie now has a new routine that is 59 feet long. Keep this distance in mind as you complete parts (a) and (b) below.

a. In his new routine, Croakie makes seven super jumps, all the same length, and then hops 3 feet. How long is each super jump?
b. If x represents the length of one super jump and $2x$ represents the length of two super jumps, write an expression that represents Croakie's routine.

4-8. Now Croakie can do a super high jump!

The first time he performed his new super-high-jump routine, he did three super-high jumps and then hopped five feet. The second time, he did only two super-high jumps and then hopped six feet. Both times, he covered the same distance. His attempts are shown in the diagram below.



a. How far does Croakie travel in one super-high jump? Explain or show how you know.
b. How long is his whole super-high-jump routine? How can you tell?

4-9. Simplify each expression below. For each expression, draw a picture or show how you know your answer makes sense.

a. $5 + (-4) + 12.65$
b. $6.5 + (-2) + 10.5$
c. $4(-5 + 100)$
d. $-212 + (-102)$
e. $4 + 6(3) + 2(5\frac{1}{2} - 1)$
f. $5 + 3(5) + (-4)(5)$

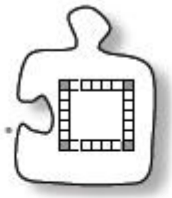
4-10. Read the Math Notes box in this lesson. Then complete the following division problems.

a. $683 \div 4$
b. $212 \div 9$

4-11. Rewrite each decimal as a fraction or fraction as a decimal.

a. 0.007 b. 0.103 c. 1.21 d. $\frac{505}{1000}$ e. $\frac{505}{100}$ f. $\frac{2}{100000}$

4.1.2 How many ways can I represent it?



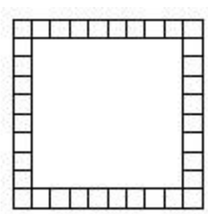
Writing Equivalent Expressions

In this lesson, you will look closely at a pattern. You will also work with your team to find different strategies for counting the number of tiles in a figure. Then you will apply your counting strategies to figures of different sizes. In your discussion, consider the questions that follow.

How do we see it?

How can we explain our thinking?

How can we describe *any* figure?



4-12. Look at the frame built with tiles at right. Then use the diagram to complete parts (a) through (d) below.

a. *Without* talking to your teammates or counting every single tile, find the number of tiles in the frame mentally. Be ready to share your method and how you see it with your team and with the class.

b. Everyone in your team is ready, share your methods one at a time. Be sure to explain to your teammates how your steps or process connect back to the drawing itself.

c. Pam told her team that when she first looked at the figure she thought that there were 40 tiles in the frame. Explain how Pam might have been looking at the drawing to see this answer and what she might have overlooked.

Lesson 4.1.2B Resource Page

d. Your teacher will now ask teams to share the methods that they discussed. Record and color-code each method on the Lesson 4.1.2B Resource Page or on the 4-12 Student eTool (CPM). As each one is presented, your teacher will demonstrate how to record and color-code it.

4-13. Below are some methods that students from another class used to find the number of tiles in problem 4-12. Which ones are like the ones that students in your class came up with? Which ones are new or different? For each new method, describe how the student might have been seeing the picture frame to come up with that method. Then add any new strategies to your resource page.

Jonas' Method: $4 \cdot 10 - 4$

Ramond's Method: $10 \cdot 10 - 8 \cdot 8$

Curran's Method: $10 + 9 + 9 + 8$

Alyssa's Method: $9 \cdot 4$

Tina's Method: $10 + 10 + 8 + 8$

TJ's Method: $4 \cdot 8 + 4$

4-14. Now imagine that the frame from problem 4-12 has been shrunk so that it is 6 tiles by 6 tiles. With your team, consider the following questions *without drawing* the frame.

a. Choose one of the methods for counting the tiles and use it to find the number of tiles in that square's frame.

b. Choose another method and use it to find the number of tiles in the 6-by-6 frame. Did you get the same answer using both methods? Should you?

4-15. Now imagine that the frame has been enlarged to be 100 tiles by 100 tiles. Choose two counting methods and use them both to find the number of tiles in the frame. Did you get the same answer using both methods? Should you?

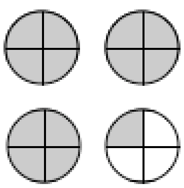
METHODS AND MEANINGS

MATH NOTES

Mixed Numbers and Fractions Greater than One

The number $3\frac{1}{4}$ is called a **mixed number** because it is composed of a whole number, 3, and a fraction, $\frac{1}{4}$.

The number $\frac{13}{4}$ is called a **fraction greater than one** because the numerator, which represents the number of equal pieces, is larger than the denominator, which represents the number of pieces in one whole, so its value is greater than one. (Sometimes such fractions are called "improper fractions," but this is just a historical term. There is nothing actually wrong with the fractions.)



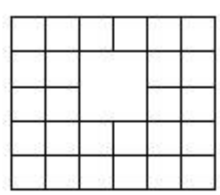
As you can see in the diagram at right, the fraction $\frac{13}{4}$ can be rewritten as $\frac{4}{4}+\frac{4}{4}+\frac{4}{4}+\frac{1}{4}$, which shows that it is equal in value to $3\frac{1}{4}$.

Your choice: Depending on which arithmetic operations you need to perform, you will choose whether to write your number as a mixed number or as a fraction greater than one.

Review & Preview

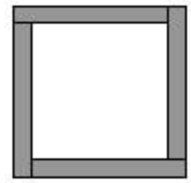
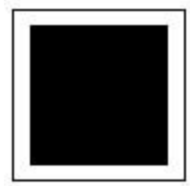
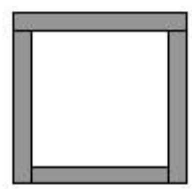
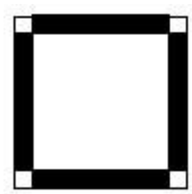
4.1.2

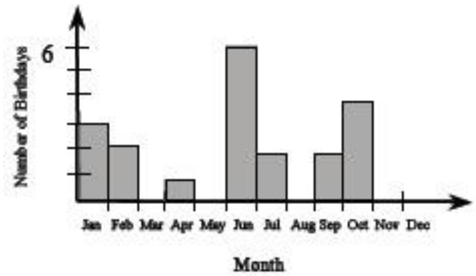
4-16. Look at the figure formed by square tiles below. How can you find out how many small squares there are in this diagram *without* counting each one? Think about this as you answer the questions below.



a. Write and simplify an expression involving addition to count the number of small squares.
b. Write and simplify an expression involving subtraction to count the number of small squares.

4-17. A team of students worked on problem 4-12. The team's work is shown below. Unfortunately, the expressions, descriptions, and diagrams got mixed up! Match the counting method, word description, and diagram that describe the same strategy.

Counting Methods	Word Descriptions	Diagrams
a. $4 \cdot 10 - 4$	1. Start in one corner and count 9 four times around the picture frame.	A. 
b. $10 + 9 + 9 + 8$	2. Take a side length of 10 four times and take away the four corners.	B. 
c. $9 \cdot 4$	3. Take the entire 100's grid and subtract the inside part of the picture frame.	C. 
d. $(10 \cdot 10) - (8 - 8)$	4. Take the top length, then add the two vertical sides and add the bottom.	D. 



4-18. Melissa collected the dates of all her friends' birthdays. The histogram above shows what she found out. Make a list of the months when her friends' birthdays occur and how many birthdays there are in each month.

4-19. Review the Math Notes box in this lesson. Then convert each mixed number to a fraction greater than one, or each fraction greater than one to a mixed number.

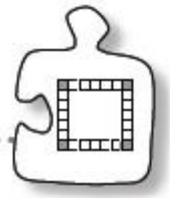
a. $4\frac{1}{8}$ b. $\frac{302}{3}$ c. $100\frac{2}{5}$ d. $\frac{18}{3}$

4-20. Each expression below begins with –5 and then adds something to it. As you look at each expression, state which direction you should move on a number line if you start at –5. Then simplify each expression.

For example, if the expression reads –5 + (–9), you would write "left, –14," since from –5 you would move *left* on a number line 9 units and would end up at –14.

a. $-5 + (-4.5)$ b. $-5 + -8$ c. $-5 + 6\frac{3}{5}$

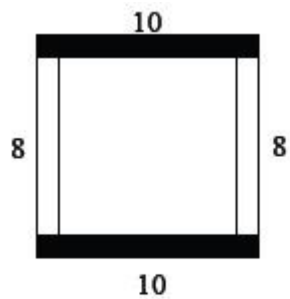
4.1.3 How can I describe *any* figure?



Using Variables to Generalize

Now it is time to use some algebra! When you understand patterns and extend your thinking to make claims that apply to *any* figure, you are using a thought process called "generalizing." Generalizing is one of the most important parts of algebraic thinking. In this lesson, you and your team will work together to generalize, using the methods you developed in Lesson 4.1.2 to describe the number of tiles in a square frame of *any* size.

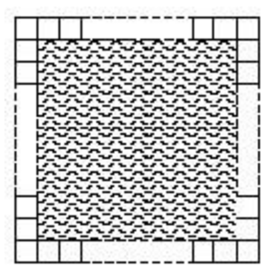
4-21. The diagram below represents one method you can use to find the number of tiles in the frame of a 10-by-10 square. Use the diagram to answer parts (a) and (b) below.



a. Look at your Lesson 4.1.2B Resource Page. Whose method was this?

b. Use this method to determine the number of tiles in the frame of a square that is 18 tiles by 18 tiles.

4-22. GENERALIZING



Can you describe how to use the method described in problem 4-21 to find the number of tiles in a square frame with *any* side length?

Your task: Work with your team to write a general set of directions in *words* that describes how to calculate the number of tiles in the frame of any square, if you are given the side length.

Discussion Points

What do we need to know to begin?

What operations or steps do we need to do?

What parts of the process change when the size of the square changes?
What parts stay the same?

4-23. What if you wanted to send the directions from problem 4-22 in a text message? When people send text messages, they often find ways to shorten words. For example, they might use a letter that sounds like a word, such as "u" instead of "you." They might also use an abbreviation like "btw" instead of "by the way."

In Lesson 4.1.1, you used a variable (such as x) to represent an *unknown* number for the distance Croakie traveled in one leap. In that case, your variable represented a number that you did not know in a specific situation. Now you are going to use a variable in a different way: to represent a number that can vary within a given situation.

Your task: You have written a set of directions for calculating the number of tiles in any square frame. How can you use numbers and symbols to shorten your directions? With your team find a way to shorten your set of directions by using a variable (such as x) to stand for "the number of tiles in one side of the frame."

4-24. Refer to your Lesson 4.1.2B Resource Page.

- Choose a *different* method from Lesson 4.1.2 for counting the number of tiles in a square frame.
- Work with your team to shorten this method into an **algebraic expression** (a combination of numbers, variables, and operation symbols).
- Be prepared to share your ideas with the class.

4-25. Compare the two expressions that you created in problems 4-23 and 4-24. The expressions both represent the number of tiles in a square frame of any side length, so they are called **equivalent expressions**. For example, there are 36 tiles in a 10-by-10 frame, no matter how you count them. For both expressions to "work," you should get the right number of tiles for any particular frame.

a. How can we check that your two expressions are equivalent?

b. Jerrold was playing around and created the following expressions for fun. Are they equivalent? How can you tell?

$2x + 2$ $\quad$ $x + 1 + x + 1$ $\quad$ $2(x + 1)$

c. Are the two expressions below equivalent?

$5 + x \cdot x \cdot x$ $\quad$ $3x + 5$

4-26. Bonnie is the owner of the "I've Been Framed!" picture-framing shop. She is excited about the work you have done describing square frames in the previous problems and now wants your help. Use your algebraic expressions to help Bonnie with each of the following orders. Be prepared to explain how you found each answer.

a. A customer wants a frame that has 8 tiles along each side. How many tiles will Bonnie need for the whole frame?

b. Bonnie's neighbor wants a frame that is 16 tiles along each side. How many tiles will she need?

c. A new customer comes into Bonnie's shop and says he wants a frame that is 25 tiles on each side. He used the expression $4(x - 1)$ to calculate that he needed 99 tiles. Bonnie explains that he actually needs only 96 total tiles. What mistake did the customer make?

d. Bonnie's father has 32 tiles that he wants to use to frame an old photograph. He needs to know the dimensions of the frame so that he can have the photo printed at the correct size. What should Bonnie tell him?

e. Bonnie has a set of 40 tiles that she bought while traveling in South Africa. What is the largest frame size (on each side) that she can make with these tiles? Will she use all of her tiles?

4-27. Bonnie has recently remodeled her "I've Been Framed!" picture-framing shop and can now make larger frames. She has just received an order for a square frame that has 102 tiles along each side. How many tiles will she need to make this frame? Explain how you got your answer.

4-28. Bonnie has been hired to make a frame to go around a large mural. She will have 300 tiles to use. How many tiles should she place along one side of the square frame for the mural? Work with your team and be prepared to describe your process to the rest of the class.

4-29. You can use the algebraic expression for a frame pattern to find the number of tiles you need to make any size of frame. The variable, which generally represents *any number*, changes to be a *specific number* when you know the side length. So you can replace the variable with that number and simplify the expression.

This process is called **evaluating** the expression for a specific value. It can be done with any algebraic expression. For example, if you know that $x = 2$ in the expression $3x + 5$, you can calculate the value of the expression by replacing the x with the number 2, writing $3(2) + 5 = 11$.

Jerrold created some more algebraic expressions for fun. Evaluate his expressions for the given value of the variable.

f. $2x + 6$ for $x = 3$

g. $25 - 3r + 2$ for $r = 8$

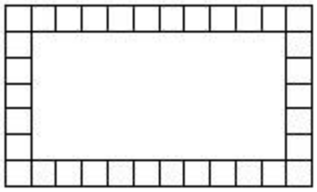
h. $4(t - 3)$ for $t = 5$

i. $4c - 12$ for $c = 5$

4-30. Bonnie's frame-shop employees, Parker and Barrow, were trying to find the total number of tiles needed for a picture frame that had 24 tiles along a side. Parker evaluated the expression

$4x - 4$ by substituting $x = 24$ into the expression, and he came up with 420 tiles. Barrow reasoned that Parker's answer was wrong. What mistake do you think Parker might have made?

4-31. When Bonnie was traveling in Bolivia, she bought 52 beautiful tiles. Sadly, when she arrived back at her frame shop, 10 of the tiles had broken. Can Bonnie make a square frame that uses all of the remaining unbroken tiles? If so, how long will the sides be? If not, what size frame could she build to use as many of her new tiles as possible?



4-32. Additional Challenge: Bonnie and her staff at "I've Been Framed!" have decided to offer a new style of picture frame. The length of the new rectangular frame is five squares longer than the width. One example of this type of frame is shown at right.

a. How many tiles make up the example frame above? Find two different ways to count.

b. If the shorter side of a frame that follows the same pattern is 10 tiles long, how long is the longer side? How do you know?

c. If the length of the shorter side is x, explain how $x + x + (x + 3) + (x + 3)$ can represent the number of tiles in the frame.

d. Show that the expression $x + x + (x + 3) + (x + 3)$ works by finding the number of tiles in a rectangular frame with a short-side length of 7.

e. Draw a diagram on your paper of one rectangular frame in this pattern. Show with arrows and colors how each part of the algebraic expression is related to the figure.

f. Could this type of frame ever be made of exactly 62 tiles? Describe how you found your answer.

4-33. LEARNING LOG

In your Learning Log use your own words to explain what a variable is. What does it mean for the value of x to change? What is an expression? Use examples with drawings to illustrate your statements. Title this entry "Variables and Expressions" and include today's date.

METHODS AND MEANINGS

MATH NOTES

Adding and Subtracting Mixed Numbers

To **add or subtract mixed numbers**, you can either add or subtract their parts, or you can change the mixed numbers into fractions greater than one.

To add or subtract mixed numbers by adding or subtracting their parts, add or subtract the whole-number parts and the fraction parts separately. Adjust if the fraction in the answer would be greater than one or less than zero. For example, the sum of $3\frac{4}{5}+1\frac{2}{3}$ is calculated below.

$$3\tfrac{4}{5} = 3 + \tfrac{4}{5} \cdot \tfrac{3}{3} = 3\tfrac{12}{15}$$
$$+1\tfrac{2}{3} = 1 + \tfrac{2}{3} \cdot \tfrac{5}{5} = +1\tfrac{10}{15}$$
$$4\tfrac{22}{15} = 5\tfrac{7}{15}$$

It is also possible to add or subtract mixed numbers by first changing them into fractions greater than one. Then add or subtract in the same way you would if they were fractions between 0 and 1. For example, the sum of $2\frac{1}{6}+1\frac{4}{5}$ is calculated below.

$$\begin{aligned} 2\tfrac{1}{6}+1\tfrac{4}{5} &= \tfrac{13}{6}+\tfrac{9}{5} \\ &= \tfrac{13}{6}\cdot\tfrac{5}{5}+\tfrac{9}{5}\cdot\tfrac{6}{6} \\ &= \tfrac{65}{30}+\tfrac{54}{30} \end{aligned}$$

4.1.3 Review & Preview

4-34. Julian was studying a pattern made with toothpicks, and he started the table shown below.

Figure Number	Number of Toothpicks
1	7
2	10
3	13
4	
5	

a. Copy and complete the table.

b. Draw axes and plot Julian's points.

c. How can you describe what all of these points have in common?

4-35. Estimate each sum or difference below by stating which whole numbers the answer should be between. Then check your conclusion by calculating the actual sum or difference.

a. $5.2 - 2.09$ b. $25\frac{1}{3} - 17\frac{5}{6}$ c. $3\frac{3}{4} + 2\frac{5}{7}$ d. $103.57 + 29.6$

4-36. Find the prime factorization for each number below.

a. 36

b. 45

c. Find the greatest common factor for 36 and 45.

d. Find the least common multiple for 36 and 45.

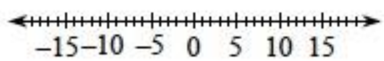
4-37. Simplify each of the following absolute value expressions.

a. $|-15|+|-26|$

b. $-|-40|$

c. $|0.5|+|-1\frac{1}{2}|$

4-38. Copy the following problems, then use the number line to help you fill in < (less than) or > (greater than) on the blank line between each pair of numbers.



a. −4.84 ___ −8.48

b. 7 ___ −7

c. -6.5 __ $-5\frac{1}{2}$

d. −1 ___0

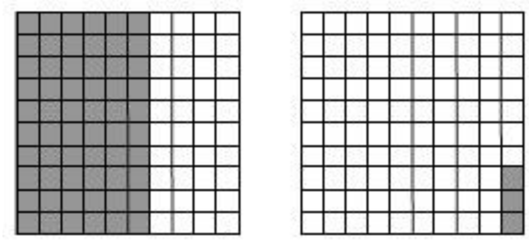
4-39. Evaluate the expressions below for the given values of the variables.

a. $6j - 3$ for $j = 4$

b. $\frac{1}{2}b + 5$ for $b = 14$

c. $8 + 4k$ for $k = 3.5$

4-40. Use the hundredths grids below to answer the following questions.



a. Give three names for the larger shaded area.

b. Give three names for the smaller shaded area.

c. What are two other names for 120%? Can you show 120% on a single hundreds grid? Explain your thinking.

4-41. Janna is training for a triathlon and wants to eat a diet with a ratio of carbohydrates to protein to fat that is 4:3:2.

a. What percent of her diet is the protein?

b. What is the ratio of carbohydrates to fat?

4-42. What is the length of the segment connecting the points (−9, 3) and (−9, −2)?

4-43. Find each sum or difference without a calculator.

a. $\frac{7}{10} + \frac{2}{3}$

b. 0.9 − 0.04

c. $3\frac{1}{4} + 2\frac{11}{12}$

d. $14\frac{1}{3} - 9\frac{1}{5}$

4.2.1 How can I enlarge a shape?

Enlarging Two-Dimensional Shapes

How do painters design murals so large that you can only see them from a distance? In most cases, designs for large projects like murals are first created as small pieces of art. Then they are **enlarged** (made bigger) to fit the space to be painted. In this lesson, you will work with your class to enlarge a design that could turn into a mural.

4-44. MYSTERY MASCOT

Jeremy and Julie are part of the spirit club at CPM Middle School. They have permission to paint a mural of their school mascot on the wall of the gym. To make it look right, they have decided to cut up a small picture of the mascot. They will then enlarge each of the pieces and put them together to form a larger model of the mural. But they need your help!

Your task: Get a piece of the original picture of the mascot and an enlargement grid from your teacher. Draw your section of the mural so that it fills the large grid yet still looks the same as the part of the original picture on your piece. Work with your team members to ensure that everyone's drawings are as accurate as possible, including the little arrow in the corner.

When all parts of the enlargements are completed, work with your class to put them together to make a paper model of the mascot mural. What is the mascot of CPM Middle School?

4-45. Your teacher will assign your team a part of the mascot to measure, such as a foot or an eye. When you have been assigned your part to measure, follow the steps below.

a. Measure your assigned part on the original mascot (the small picture) in centimeters and then on the corresponding (identical) part of the enlarged mascot.

b. Work with your class to share data and complete a table like the one below.

mascot part	original (cm)	enlarged (cm)

c. With your team, examine the data collected by your class. Look for a way to describe the relationship between parts of the original mascot picture and the enlarged model of the mascot. Be prepared to share your ideas with the class.

d. Is there any part of the enlarged picture that seems to be the wrong size? How could you check?

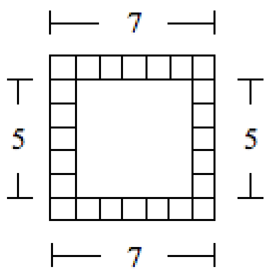
4-46. Why is it necessary that all parts of the original picture grow in the same way? What if, for example, the nose got two times bigger and the eyes got five times bigger? Work with your team to explain what has to happen for the mascot model to keep its shape as it gets larger.

METHODS AND MEANINGS

MATH NOTES

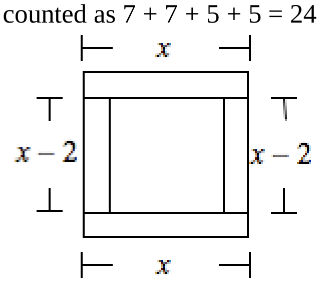
Using Variable to Generalize

Variables are letters or symbols used to represent one or more numbers. They are often used to generalize patterns from a few specific numbers to include all possible numbers.



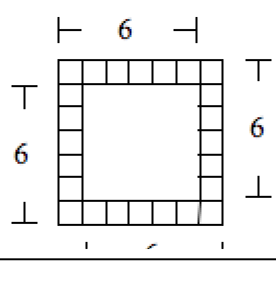
For example, if a square is surrounded by smaller square tiles each measuring one centimeter on a side, how many tiles are needed? It helps to look at a specific size of square first.

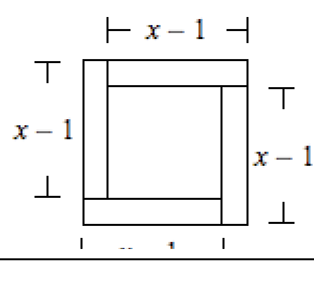
The outside square below has side length 7. One way to see the total number of tiles needed for the frame is to consider that it needs 7 tiles for each of the top and bottom sides and $7 - 2 = 5$ tiles for the left and right sides. This is shown in the first diagram at right. The total number of tiles needed for the frame can be counted as $7 + 7 + 5 + 5 = 24$.



Square frames with different side lengths will follow the same pattern. You can generalize by writing an expression for any side length, denoted by x. The second diagram at right shows that the top and bottom each contain x tiles. The right and left sides each contain $x - 2$ tiles. You could write the total number of tiles as either $x + x + (x - 2) + (x - 2)$ or $2 \cdot x + 2 \cdot (x - 2)$ or even as $4x - 4$.

Shown below are two additional square-frame diagrams. The diagram on the left shows another way to count the number of tiles in a frame. The diagram on the right shows the algebraic expression associated with it. Notice that the expression resulting from this counting method could be written $(x - 1) + (x - 1) + (x - 1) + (x - 1)$, or it could be written $4 \cdot (x - 1)$.





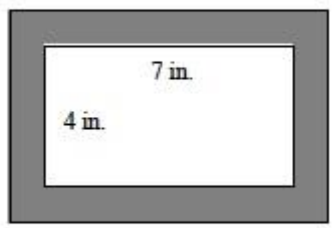
4.2.1

4-47. Use graph paper to complete the steps below. Then answer the question that follows. Draw a square that measures 5 units on each side.

- Draw a design inside your 5 × 5 square.
- Then draw a square that measures 15 units on each side.
- Enlarge your picture as accurately as possible so that it fits inside of the 15 × 15 square.

How much wider and how much longer is your new picture?

4-48. Tina is going to put 1-inch square tiles on the picture frame shown below.

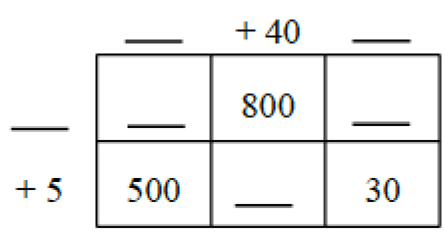


d. If the frame is one tile wide, how many 1-inch-square tiles will she need?

e. Would more 1-inch square tiles fit inside the frame or on the frame? Show how you know.

4-49. Four friends worked together to wash all of the cars that the Kumar family owns. They received $43.00 for doing the work and agreed to divide the earnings evenly. How much money will each friend earn? Show how you know.

4-50. Copy and complete the generic rectangle below. What multiplication problem does it represent and what is the product?



4-51. Use the portions representation web to rewrite each percent as a fraction, as a decimal and with words or a picture.

a. 13% b. 20% c. 130% d. 32%

4.2.2 How does it change?

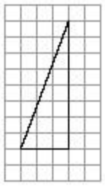
Enlarging and Reducing Figures

As you learned from enlarging the CPM Middle School mascot in Lesson 4.2.1, an image is enlarged correctly and keeps its shape when all measurements grow the same way. Shapes that are correct **enlargements** (larger versions) or **reductions** (smaller versions) of each other are called **similar**. In this lesson, you will consider what it means mathematically for all parts of a shape to grow or shrink in the same way.

4-52. THE BROKEN COPIER

The Social Studies teachers at CPM Middle School are working together to plan a geography unit. They are using all of the school's copy machines to **enlarge** (make larger) and **reduce** (make smaller) images from books to make them convenient sizes. The teachers think that some of the copy machines might be broken and are making incorrect copies.

Your task: Get the Lesson 4.2.2 Resource Page from your teacher. Work with your team to identify which, if any, of the images have been made using a broken copier. Be ready to explain how you can tell if any of the copies are incorrect.



4-53. Carmen and Dolores want to enlarge the triangle at left. Its base is three units long. They want the base of their new triangle to be 12 units long, and they want the shape of the new triangle to stay the same. However, they disagree about what the new triangle's height should be.

a. Work with your team to predict the height of the new triangle.

b. Carmen noticed that the new base is 9 units longer than the original one, so she thinks that the height of the new triangle should be 9 units longer, or 17 units high. Dolores noticed that the new base is 4 times longer, so she thinks that the height of the new triangle should be 4 times longer, or 32 units high.

i. On graph paper, draw the original triangle as well as the triangles that Carmen and Dolores describe.

ii. Who is correct? How can you tell?

c. What if Carmen and Dolores wanted to reduce the shape so that the base of the new smaller triangle is 1 unit long? How tall should the triangle be to keep its original shape? How did you figure this out? Draw the new shape on your graph paper.

4-54. Since some of the copiers at CPM Middle School are broken, the math teachers plan to do all of their reductions and enlargements by hand. They need your team's help.

Using graph paper, draw each of the original figures described in parts (a) and (b) below and enlarge or reduce them as described.

a. Draw a rectangle that measures 5 units by 3 units. Enlarge it so that each side is four times as long as the original.

b. Draw a right triangle with a base of 2 units and a height of 3 units. Make three "copies" so that the lengths of the new sides are 50%, 300%, and 500% of the original.

4-55. Draw a coordinate grid with four quadrants. Label the *x*- and *y*-axes from –10 to 10 and then use it to do the following tasks.

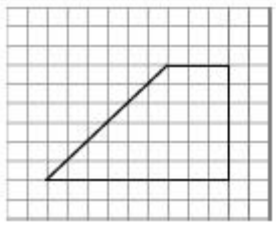
a. Plot the following ordered pairs and connect them. (−2, −4), (−2, 4), (2, 4) and (2, −4). What is the shape that you have made?

b. What is the length of each of the sides of the shape that you have made?

c. Draw a figure that is enlarged by a factor of 1.5 and still has one corner (or **vertex**) at (–2, –4). What are the coordinates of the corners (or **vertices**) for the new shape? What are the lengths of the sides now?

d. Now draw a figure that is $\frac{3}{4}$ the size of the original, again with one vertex still at (–2, –4). What are the coordinates of the vertices of the reduced shape? What are its side lengths?

Graph Paper



4-56. Additional Challenge: Copy the diagram above onto graph paper. Then draw a smaller copy with sides that are $\frac{2}{3}$ the lengths of the original.

4-57. LEARNING LOG

Work with your team to describe how you can tell if an image has been enlarged correctly. When you have come to an agreement, write your ideas as a Learning Log entry. Title this entry “Enlarging Figures” and label it with today’s date.

METHODS AND MEANINGS

MATH NOTES

Evaluating Algebraic Expressions

An **algebraic expression**, also known as a *variable expression*, is a combination of numbers and variables, connected by mathematical operations. For example, $4x$, $3(x – 5)$, and $4x – 7$ are algebraic expressions.

Addition and subtraction separate expressions into parts called **terms**. For example the expression above, $4x – 3y + 7$, has three terms: $4x$, $−3y$, and 7.

A more complex expression is $2x + 3(5 – 2x) + 8$. It also has three terms: $2x$, $3(5 – 2x)$, and 8. But the term $3(5 – 2x)$ has another expression, $5 – 2x$, inside the parentheses. The terms of this inner expression are 5 and $–2x$.

To **evaluate** an algebraic expression for particular values of variables, replace the variables in the expression with their known numerical values and simplify. Replacing the variables with their known values is called **substitution**. An example is provided below.

Evaluate $4x – 3y + 7$ for $x = 2$ and $y = 1$. Replace x and y with their known values of 2 and 1, respectively, simplify.

$$4(2)-3(1)+7$$

$$8-3+7$$

$$12$$

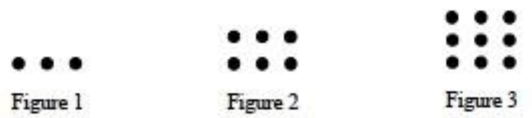
4.2.2 Review & Preview

4-58. Draw two different simple geometric shapes (such as rectangles or right triangles) on graph paper.

a. Choose one shape and enlarge it so that each side is twice as long as the original.

b. Choose the other shape and reduce it so that each side is half the length of the original.

4-59. Study the pattern below. Sketch and label the fourth and fifth figures. Then predict how many dots will be in the 100th figure. Write an expression you can use to determine the number of dots in any figure.



4-60. Simplify each of the following absolute value expressions.

a. $|-25.6|+|-11.4|$

b. $-\left|-3\frac{2}{7}\right|$

c. $|0.375|+\left|-\frac{5}{8}\right|$

4-61. Compute each sum or difference.

a. $\frac{2}{3}+\frac{1}{5}$

b. $\frac{7}{8}-\frac{1}{4}$

c. $1\frac{2}{3}+3\frac{1}{4}$

d. $7-3\frac{2}{5}$

4-62. Find each quotient without using a calculator.

a. 42.5 ÷ 1.5

b. 589.2 ÷ 16

c. 5 ÷ 9

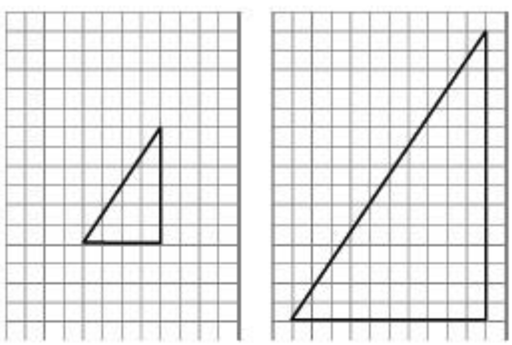
4.2.3 How can I compare them?

Enlargement and Reduction Ratios

In the past few lessons, you enlarged and reduced images while preserving their shapes. By doing so, you created **similar** figures. You learned, for example, that to enlarge a shape to 300% of the original, you multiplied the length of each side by 3.

If you want to compare side lengths of similar figures, one way to do so is by using **ratios**. A ratio compares lengths by dividing. In this lesson, you will learn about using ratios to determine whether enlargements or reductions were done correctly. As you work with your team, use the questions below to help start your discussions.

- How does the shape change?
- What are we comparing?
- How can we describe the relationship?



4-63. Andrew, Barb, Carlos, and Dolores were looking at the similar triangles at right. "Similar" in this context means that the triangles have the same shape, but they are different sizes. Lesson 4.2.3 Resource Page

"I'm confused," said Carlos. "Is the triangle on the right an enlargement of the triangle on the left, or is the triangle on the left a reduction of the triangle on the right?"

a. Work with your team to find a way to describe the relationship between the lengths of the sides of these two triangles. Think about how each triangle might have been created from the other one. Be prepared to share your ideas with the class.

b. "Hey," Barb said, "I just learned about ratios from my sister. She told me that ratios are another way to compare quantities like the dimensions of these triangles. We could compare these triangles by setting up the ratio of 4 units to 10 units. We can write it in these ways."

4 : 10

$\frac{4}{10}$

4 to 10

Carlos wondered, *"But wait, why wouldn't the ratio be 6 to 15?"*

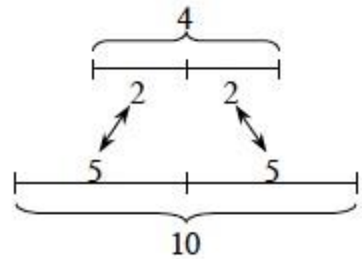
i. Where did Barb and Carlos get the numbers that they are using in their ratios? What are they comparing?

ii. Whose ratio is correct? How do you know?

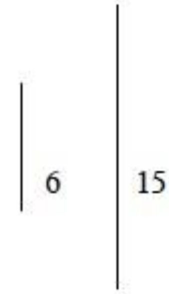
iii. What are some other ratios that represent the same relationship as 4:10? Work with your team to find at least three other ratios and be prepared to share them with the class.

c. Dolores was confused and wondered, *"Why isn't the ratio 10:4?"* What do you think?

4-64. Andrew had a new idea. He drew the diagram at right and described the relationship between the triangles with the ratio 2:5.



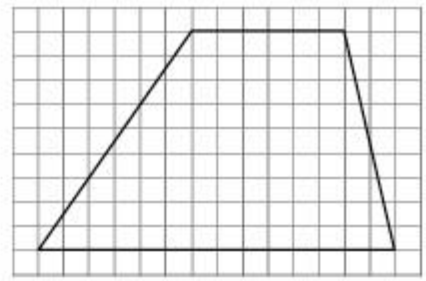
a. Is the ratio 2:5 the same as Barb's ratio of 4:10? Why or why not?



b. Dolores drew the heights of the two triangles, as shown at right. How could she see the ratio of 2:5 in this diagram? Discuss this with your team and be prepared to explain your ideas to the class.

4-65. You may remember that a quadrilateral is a four-sided figure. There are many different kinds of quadrilaterals. On graph paper, draw a quadrilateral with one side that is 12 units long and another side that is 9 units long. Then reduce your quadrilateral so that the ratio of sides of new to original is 2 to 3.

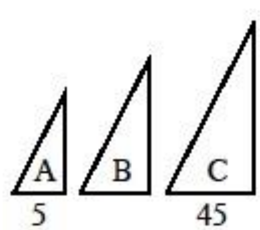
4-66. Xenia drew the trapezoid shown below. She wants to draw another figure of the same shape so that the relationship between the two figures can be described by a ratio of 2 to 7 or $\frac{2}{7}$. What will be the length of the longest side of her new shape? Is there more than one possibility for her new shape?



4-67. TEAM CHALLENGE

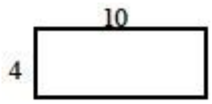
Carlos was working with his team to solve the following challenge problem.

Triangles A, B, and C are shown below. The ratio of the sides of triangle A to triangle B is the same as the ratio of the sides of triangle B to triangle C.



Carlos says that the base of triangle B must be 25 units long, because 25 is halfway between 5 and 45. Is he correct? If so, explain how you can be sure. If not, what *is* the length of the base of triangle B? How do you know?

4-68. You have discovered that when you enlarge a figure, the ratio of side lengths between the original and the enlargement stay the same. What about the perimeters? What about the areas? When you enlarge a figure, does the ratio of the *perimeters* between the original and the enlargement stay the same, too? What about the ratio of the *areas?* Think about this as you conduct the following investigation.

a. 

Find the perimeter and area of the rectangle at right.

a. Draw a new rectangle that is an enlargement of the rectangle at right, so that the ratio of the sides of the original rectangle to the new one is 2:3. Label the length and width.

b. Find the perimeter and area of the new, larger rectangle.

c. Write the ratio of the original perimeter to the new perimeter. Then write the ratio of the original area to the new area. Are the ratios the same?

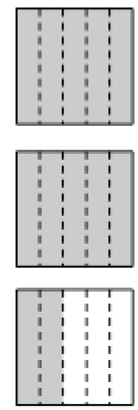
4-69. LEARNING LOG

Today you learned about a different way to compare similar figures, by using a ratio. Write a Learning Log entry describing what you know so far about ratios. Include the different ways that ratios can be written. Also include an example of how you can use a ratio to enlarge or reduce a figure. Title this entry "Ratios" and label it with today's date.

4.2.3 Review & Preview

4-70. On graph paper, draw any quadrilateral. Then enlarge (or reduce) it by each of the following ratios.

a. $\frac{4}{1}$ b. $\frac{7}{2}$



4-71. George drew the diagram at right to represent the number $2\frac{2}{5}$.

"Look," said Helena, "This is the same thing as $\frac{12}{5}$." What do you think? Explore this idea in parts (a) through (c) below.

a. Is Helena correct? If so, explain how she can tell that the diagram represents $\frac{12}{5}$. If she is not correct, explain why not.

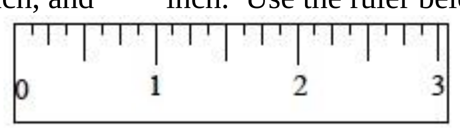
b. Draw a diagram to represent the mixed number $3\frac{2}{3}$. How can you write this as a single fraction greater than one?

c. How can you write $\frac{7}{4}$ as a mixed number? Be sure to include a diagram in your answer.

4-72. Simplify the following expressions.

a. $1\frac{1}{2}+2\frac{1}{8}$ b. $\frac{4}{5}-\frac{2}{3}+\frac{1}{6}$ c. $5\frac{3}{5}-1\frac{4}{5}$

4-73. A new shipment of nails is due any day at Hannah's Hardware Haven, and you have been asked to help label the shelves so that the nails are organized in length from least to greatest. She is expecting nails of the following sizes: $1\frac{3}{8}$ inch, $1\frac{7}{8}$ inch, $2\frac{1}{4}$ inch, $\frac{7}{8}$ inch, and $1\frac{1}{2}$ inch. Use the ruler below to help Hannah order the labels on the shelves from least to greatest.



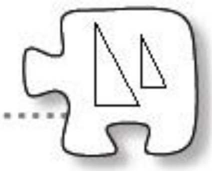
4-74. Cecelia wants to measure the area of her bedroom floor. Should she use square inches or square feet? Complete parts (a) through (c) below as you explore this question.

a. Write a sentence to explain which units you think Cecelia should use.

b. If Cecelia's bedroom is 12 feet by 15.5 feet, what is the area of the bedroom floor? Show how you got your answer.

c. Find the perimeter of Cecelia's bedroom floor. Show how you got your answer.

4.2.4 How can I use ratios?



Ratios in Other Contexts

Are ratios only used to compare shapes that have been enlarged or reduced? In this lesson, you will expand your use of ratios to new situations. *"What is being compared?"* is a question that will be useful to keep in mind as you work with your team on this lesson.

4-75. Katura was making berry drink from a bag of powdered mix. The directions said to use 5 scoops of the powder for every 8 cups of water. Her pitcher holds 12 cups of water.

a. What is the ratio of powder to water in the directions?

b. Work with your team to figure out how much powder Katura needs to mix with 12 cups of water. Try to find more than one way to describe or show how you know that your answer makes sense. Be prepared to explain your ideas to the class.

c. What is the ratio of powder to water in Katura's pitcher? How does this compare to the ratio in the directions?

4-76. ON THE TRAIL AGAIN

Ms. Hartley's students were working with their mix of raisins and peanuts from Chapters 1 and 2. The class found that 30% of the mix was raisins. Sophie was working with a sample from the mix and counted 42 peanuts in it.

Sophie had just poured her sample back into the jar, when she realized that she had counted the wrong thing! Her teacher wanted to know how many *raisins* were in the sample, not *peanuts*! Work with your team and use the questions below to help Sophie figure out a reasonable estimate of how many raisins were in her sample.

a. Sophie knows that 30% is the same as $\frac{30}{100}$. Can this be thought of as a ratio? Which two quantities are being compared in this case? Can you write another equivalent ratio representing the same comparison?

b. Could Sophie write a ratio comparing the number of raisins to peanuts? How could you figure out this ratio without having to count the peanuts? Discuss this with your team and be ready to explain your thinking to the class.

c. Find an equivalent ratio that will help Sophie figure out how many raisins should have been in her sample that contained 42 peanuts.

4-77. Nicci is setting up a carnival machine with 3 teddy bears, 7 stuffed frogs, 3 rubber duckies, and 2 stuffed dinosaurs.

a. Find the following ratios for Nicci's machine:

i. The number of teddy bears to total prizes.

ii. The number of teddy bears to the number of stuffed dinosaurs.

iii. The number of teddy bears to the combined number of other prizes.

b. In the carnival game, one prize is chosen at random. Nicci's teacher told her that the probability of randomly picking a teddy bear was 20%. Which of the ratios in part (a) do you think her teacher used to find the probability?

c. Nicci is setting up a different machine that holds 60 total prizes. The machine will have the same ratios for each kind of prize as her first machine. If the new machine has 12 teddy bears, will the chances of randomly picking a teddy bear be the same as for her original machine? Explain.

4-78. Trei correctly spelled 60% of the words on her last spelling test!

a. How many words did she spell correctly for each word that she spelled wrong? That is, what is her ratio of correctly to incorrectly spelled words?

b. Luis spelled 3 words correctly for every 1 that he spelled incorrectly. Did Luis do better than Trei on the test? What is Luis's score represented as a percent?

4-79. Additional Challenge: A box is filled with green marbles, red marbles, and blue marbles. The ratio of red marbles to green marbles is 3:1. The ratio of green marbles to all of the marbles in the box is 2:11. Write each of the following ratios.

a. The ratio of red marbles to the total number of marbles.

b. The ratio of blue marbles to the total number of marbles.

c. The ratio of blue marbles to green marbles.

d. The ratio of red marbles to blue marbles.

METHODS AND MEANINGS

MATH NOTES

Ratios

A **ratio** is a comparison of two numbers, often written as a quotient; that is, the first number is divided by the second number (but not zero). A ratio can be written in words, in fraction form, or with colon notation. Most often, in this class, you will either write ratios in the form of fractions or state the ratios in words.

For example, if there are 38 students in the school band and 16 of them are boys, we can write the ratio of the number of boys to the number of girls as:

16 boys to 22 girls $\quad \frac{16 \text{ boys}}{22 \text{ girls}} \quad$ 16 boys : 22 girls

Review & Preview

4.2.4

4-80. Richie and Bethany play basketball and practice shooting free throws after school. During one practice session, Richie attempted 15 free throws and made 12 of them.

a. Write a ratio comparing the number of free throws he made to the number that he missed.

b. Bethany made eight free throws for every three that she missed. Did Bethany do better than Richie? Show how you know.

4-81. This problem is a checkpoint for addition and subtraction of mixed numbers. It will be referred to as Checkpoint 4.

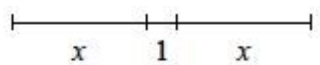
Compute each sum or difference. Simplify if possible.

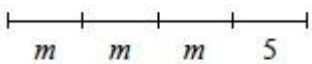
a. $5\frac{1}{2}+4\frac{2}{3}$ b. $1\frac{5}{6}+2\frac{1}{5}$ c. $9\frac{1}{3}-4\frac{1}{5}$ d. $10-8\frac{2}{3}$

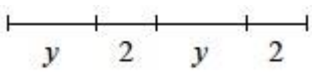
Check your answers by referring to the Checkpoint 4 materials.

Ideally, at this point you are comfortable working with these types of problems and can solve them correctly. If you feel that you need more confidence when solving these types of problems, then review the Checkpoint 4 materials and try the practice problems provided. From this point on, you will be expected to do problems like these correctly and with confidence.

4-82. Use an algebraic expression to represent each sequence of lengths shown below.

a. 

b. 

c. 

4-83. In parts (a) through (c) below, refer to the previous problem. You will find the length of the line segments in problem 4-82 by substituting given values for the variables. For example, if x is 3 units in part (a) of problem 4-82, the line segment would be $3 + 1 + 3 = 7$ units long.

a. Find the length of the line segment in part (a) of problem 4-82 using $x = 4\frac{1}{2}$

b. Find the length of the line segment in part (b) of problem 4-82 $m = 4$

c. Find the length of the line segment in part (c) of problem 4-82 using $y = 5.5$

4-84. Write each fraction greater than one as a mixed number and each mixed number as a fraction greater than one.

a. $5\frac{8}{19}$ b. $\frac{17}{8}$ c. $7\frac{7}{15}$ d. $\frac{19}{5}$

Chapter 4 Closure What have I learned?

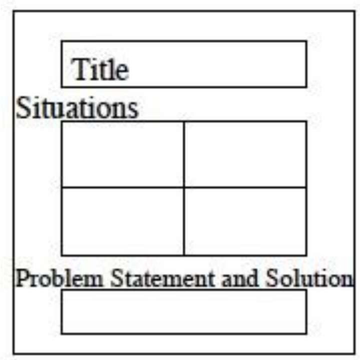
Reflection and Synthesis

The activities below offer you a chance to reflect about what you have learned during this chapter. As you work, look for concepts that you feel very comfortable with, ideas that you would like to learn more about, and topics you need more help with.

1. SUMMARIZING MY UNDERSTANDING

This section gives you an opportunity to show your understanding of how to enlarge and reduce figures and how to use ratios, two of the main ideas of this chapter.

Team Poster



You have learned many things in this chapter: how to enlarge and reduce figures while maintaining their shapes, how to use ratios to describe relationships between shapes of different sizes, how to use ratios in other contexts, and how to find the value of an unknown variable in a specific situation involving a ratio. This section gives you an opportunity to demonstrate what you know so far about these concepts. Today you and your team will create a poster that illustrates the skills and knowledge that you have developed in these areas.

Brainstorm Situations: Follow your teacher's instructions to brainstorm a list of different situations where a ratio could be used to answer a question.

Situation Descriptions: Work with your team to think of four different situations for which a ratio could be used. Then each person should write a description of one of the situations and suggest a ratio to use for the situation. Be sure to provide enough information so that someone unfamiliar with the situation would understand what you mean.

Write a Problem: Follow your teacher's instructions to select one situation randomly. Then work with your team to use that situation to write a problem. Remember that you will need to provide all of the necessary information and details for someone else to be able to solve the problem. Show your problem to your teacher before the next step.

Solve Your Problem: Now your team should find the answer to your problem. This should include writing a ratio and then showing how to get the answer. Be sure to include your reasoning for your process and enough of your steps that anyone looking at them will know what you did.

Team Poster: Follow the model above to label and construct the sections of your poster from the pieces that your team has created. Decide together on a creative title for your poster.

2. WHAT HAVE I LEARNED?

Doing the problems in this section will help you to evaluate which types of problems you feel comfortable with and which ones you need more help with.

Solve each problem as completely as you can. The table at the end of this closure section provides answers to these problems. It also tells you where you can find additional help and where to find practice problems like them.

CL 4-85. Draw a number line and place a point for each of the following portions on it.

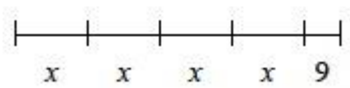
a. $\frac{4}{5}$ b. 0.003 c.30% d. $\frac{7}{6}$ e.0.75 f. $\frac{3}{7}$ g. $\frac{1}{3}$ h. $\frac{112}{112}$

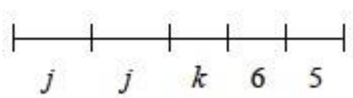
CL 4-86. Evaluate the following algebraic expressions.

a. Find the value of $7m + 9$ for $m = 2$.

b. Find the value of $a \cdot b$ for $a = 10$ and $b = 4$.

CL 4-87. Write an expression to represent the length of each of the ropes shown below. Then find the length of each rope if $x = 20$, $j = 10$, and $k = 7$.

a. 

b. 

CL 4-88. Simplify each expression.

a. $|15|+|-1|$ b. $|6|+|0|$ c. $-|2|+|8|$

CL 4-89. Copy the dot pattern below and draw Figures 0, 4, and 7. Write an expression to describe how the pattern is growing.

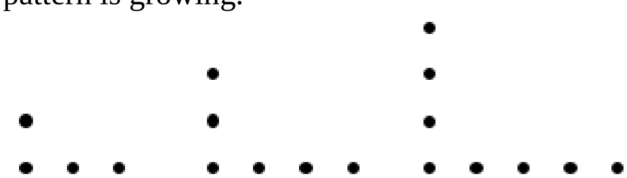


Figure 1 Figure 2 Figure 3

CL 4-90. Draw a right triangle on graph paper that has a base of 4 units and a height of 2 units. Enlarge it so that each side is 2.5 times as long as the original.

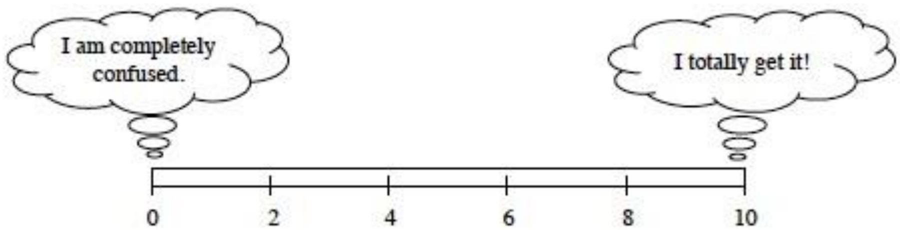
CL 4-91. Describe how each of the following enlargement or reduction ratios would change the size of a photograph. The given ratios are from the new figure to the original figure.

a. $\frac{15}{2}$ b. $\frac{4}{3}$ c. $\frac{5}{6}$ d. $\frac{12}{12}$

CL 4-92. Use a coordinate grid to plot the points (−2, 3) and (4, 5). Then plot two more points so that all four points form vertices of a rectangle with a horizontal length. Next, find the length of each side. Write an absolute value expression to show how you calculated each length.

CL 4-93. For each of the problems above, do the following:

Draw a bar or number line that represents 0 to 10.



Color or shade in a portion of the bar that represents your level of understanding and comfort with completing that problem on your own.

If any of your bars are less than a 5, choose *one* of those problems and do one of the following tasks:

Write two questions that you would like to ask about that problem.

Brainstorm two things that you DO know about that type of problem.

If all of your bars are at 5 or above, choose one problem and do one of these tasks:

Write two questions you might ask or hints you might give to a student who was stuck on the problem.

Make a new problem that is similar and more challenging than that problem and solve it.

3. WHAT TOOLS CAN I USE?

You have several tools and references available to help support your learning: your teacher, your study team, your math book, and your Toolkit, to name only a few. At the end of each chapter, you will have an opportunity to review your Toolkit for completeness. You will also revise or update it to reflect your current understanding of big ideas.

The main elements of your Toolkit should be your Learning Log, Math Notes, and the vocabulary used in this chapter. Math words that are new appear in bold in the text. Refer to the lists provided below and follow your teacher's instructions to revise your Toolkit, which will help make it useful for you as you complete this chapter and as you work in future chapters.

Learning Log Entries

Lesson 4.1.3 - Variable

Lesson 4.2.2 - Enlarging Figures

Lesson 4.2.3 - Ratios

Math Notes

Lesson 4.1.1 - Dividing

Lesson 4.1.2 - Mixed Numbers and Fractions Greater than One

Lesson 4.1.3 - Adding and Subtracting Mixed Numbers

Lesson 4.2.1 - Using Variables to Generalize

Lesson 4.2.2 - Evaluating Algebraic Expressions

Lesson 4.2.4 - Ratios

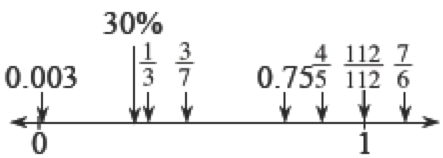
Mathematical Vocabulary

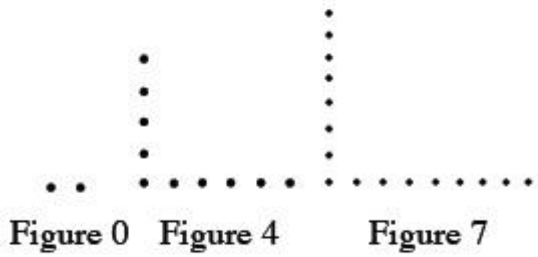
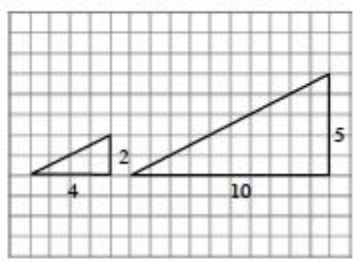
The following is a list of vocabulary found in this chapter. Some of the words have been seen in a previous chapter. The *italicized* words are new to this chapter. Make sure that you are familiar with the terms below and know what they mean. Click on the word for a "pop-up" definition. For more information, refer to the glossary or index. You might also add these words to your Toolkit so that you can reference them in the future.

algebraic expression	***enlarge***	***equivalent expressions***
equivalent fractions	**equivalent ratios**	**expression**
evaluate	**ratio**	***reduce***
similar figures	***substitution***	***variable***
vertex (vertices)		

Answers and Support for Closure Problems
What Have I Learned?

Note: MN = Math Note, LL = Learning Log

Problem	Solution	Need Help?	More Practice
CL 4-85.		Section 3.1 MN: 3.1.5 LL: 3.1.4 and 3.1.5	Problem CL 3-138 and 4-74
CL 4-86.	a. $7(2) + 9 = 14 + 9 = 23$ b. $10 \cdot 4 = 40$	Section 4.1 MN: 4.2.1	Problems 4-29, 4-39, and 4-83
CL 4-87.	c. $x + x + x + x + 9$ or $4x + 9$; 89 d. $j + j + k + 11$ or $2j + k + 11$; 38	Section 4.1 MN: 4.2.2 LL: 4.1.3	Problems 4-7, 4-29, 4-39, and 4-82
CL 4-88.	e. 16 f. 6 g. 6	Lessons 3.2.3 and 3.2.4 MN: 3.2.4 LL: 3.2.3	Problems 3-128, 4-37, 4-42, and 4-60
CL 4-89.		Lesson 1.1.3	Problems CL 1-95, CL 2-92, 3-20, and 4-59

	 Two dots are added to each figure: one on the far right and one on the top. $(n + 2) + n$		
CL 4-90.	 	Section 4.2 LL: 4.2.2	Problems 4-47, 4-58, and 4-70
CL 4-91.	h. Each of the sides would get a lot (more than 7 times) longer. i. Each of the sides would get a little bit longer. j. Each of the sides would get a little bit shorter. k. Each of the sides would stay exactly the same length.	Section 4.2 LL: 4.2.2 and 4.2.3	Problems 4-54 and 4-70
CL 4-92.	Points: (−2, 5) and (4, 3) Length: $\lvert -2 \rvert + \lvert 4 \rvert = 6$ units Width: $\lvert 5 \rvert - \lvert 3 \rvert = 2$ units	Lessons 3.2.3 and 3.2.4 MN: 3.2.4 LL: 3.2.3	Problems 3-128, 3-129, 4-42, and 4-60